Close today: HW_2A, 2B (5.3,5.4)
Closes tomorrow: HW_2C (5.5)

5.5 The Substitution Rule

The Substitution Rule:
If we write u = g(x) and du = g'(x) dx, then

$$\int f(g(x))g'(x)dx = \int f(u)du$$

Observations from last class
1. We are reversing the "chain rule".
2. In each case, we see
   "inside" = a function inside another
   "outside" = derivative of inside

Entry Task:
Let $u = 1 + x^4$ and correctly change the variable in

$$\int x^3(1 + x^4)^{11}\, dx$$

2. $\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}}\,dx$

Aside: What is really happening
(you do not need to write this)

Recall:

$$\int_a^b f(g(x))g'(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(g(x_i))g'(x_i)\Delta x$$

If we replace u = g(x), then we are "transforming" the problem from one involving *x* and *y* to one with *u* and *y*.

This changes ***everything*** in the set up. The lower bound, upper bound, widths, and integrand all change!

Recall from Math 124 that

$$g'(x) = \frac{du}{dx} \approx \frac{\Delta u}{\Delta x}$$

(with more accuracy when Δx is small)

Thus, we can say that

$$g'(x)\Delta x \approx \Delta u$$

In other words, if the width of the rectangles using *x* and *y* is Δx, then the width of the rectangles using *u* and *y* is $g'(x)\Delta x$.

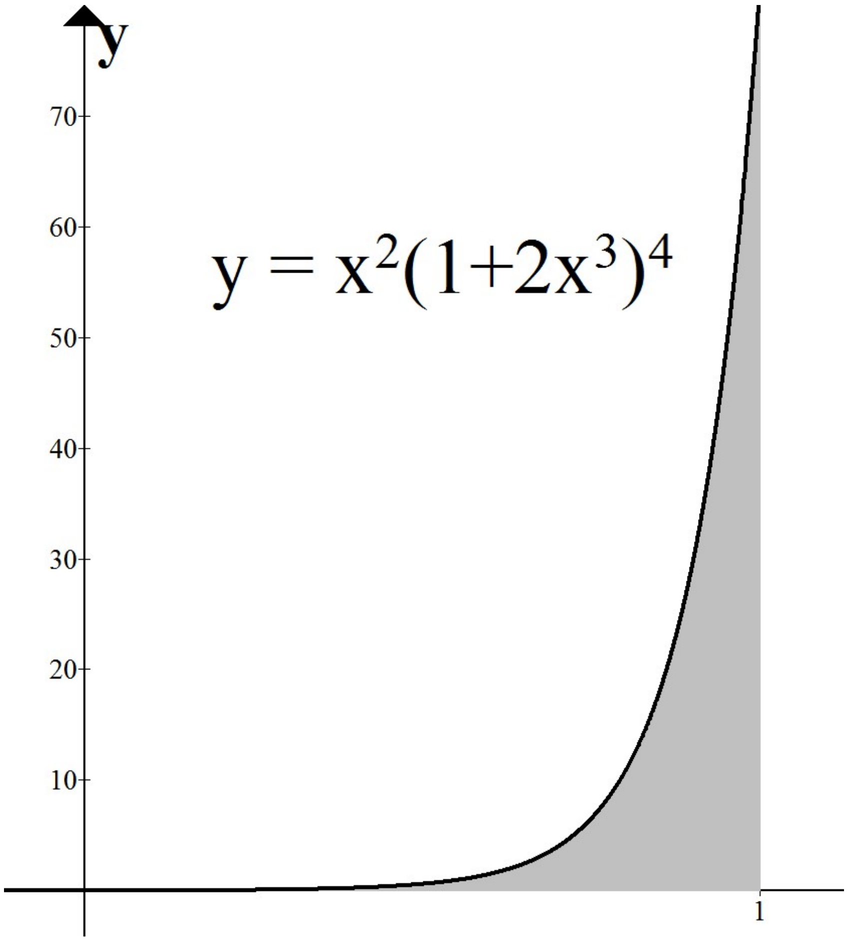
And if we write $u_i = g(x_i)$, then

$$\int_a^b f(g(x))g'(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(g(x_i))g'(x_i)\Delta x$$

$$= \lim_{n\to\infty}\sum_{i=1}^{n} f(u_i)\Delta u$$

$$= \int_{g(a)}^{g(b)} f(u)du$$

Here is a visual example of this transformation

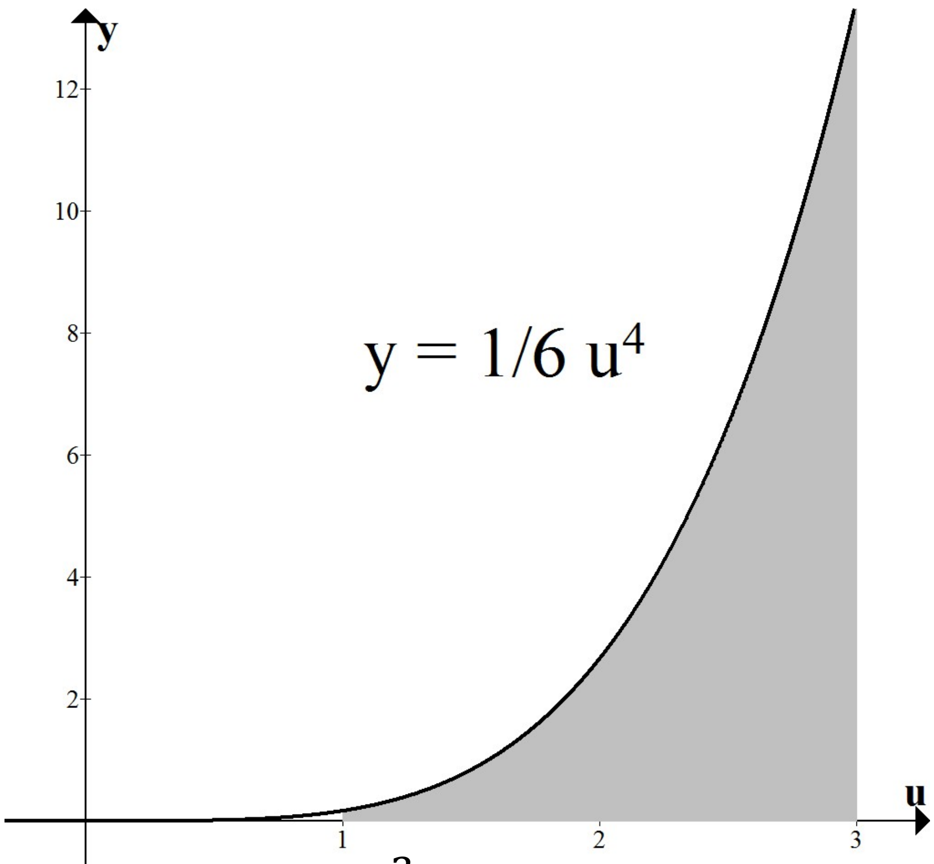


$$\int_0^1 x^2(1+2x^3)^4 dx$$

Let $u = 1 + 2x^3$.

Change *everything* in terms of *u.*

We get



$$\int_1^3 \frac{1}{6} u^4 du$$

Example: Evaluate

$$\int_{2}^{3} x^2 e^{x^3}\, dx$$

Example: Compute

$$\int_2^3 x^2 e^{x^3}\, dx$$

Advice on picking u = ???

Try u = *inside*

Try u = *denominator*

It doesn't take long to try something, so experiment!

$$\int_0^1 \frac{x}{x^2+3}\,dx$$

$$\int \mathrm{x}\cos(\sin(x^2))\cos(x\text{^}2)\,dx$$

$$\int \tan(x)\, dx$$

What to do when the “old” variable remains:

Examples:

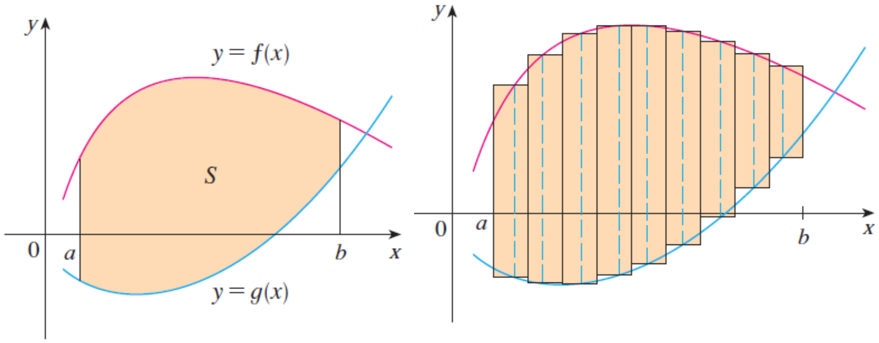
1. $\int x^3\sqrt{2 + x^2}\, dx$

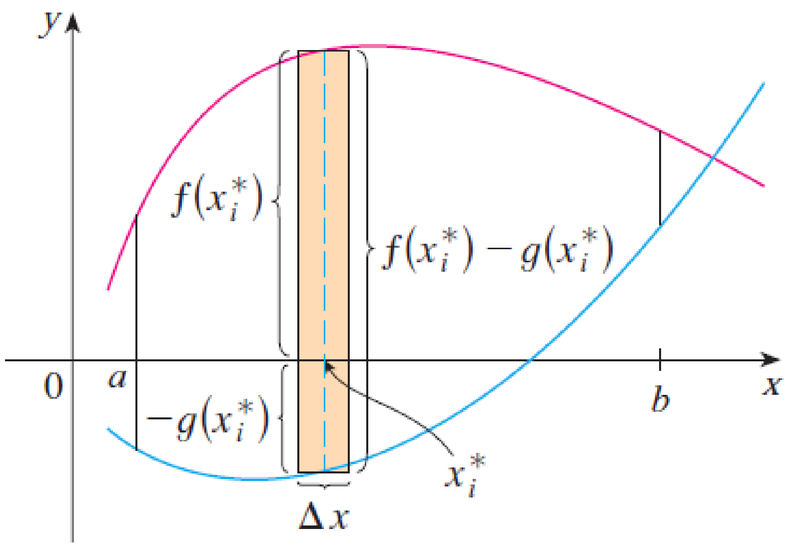
2. $\int \frac{\mathrm{x}^7}{x^4 + 1} dx$

Ch 6: Basic Integral Applications

6.1 Areas Between Curves

Using dx:

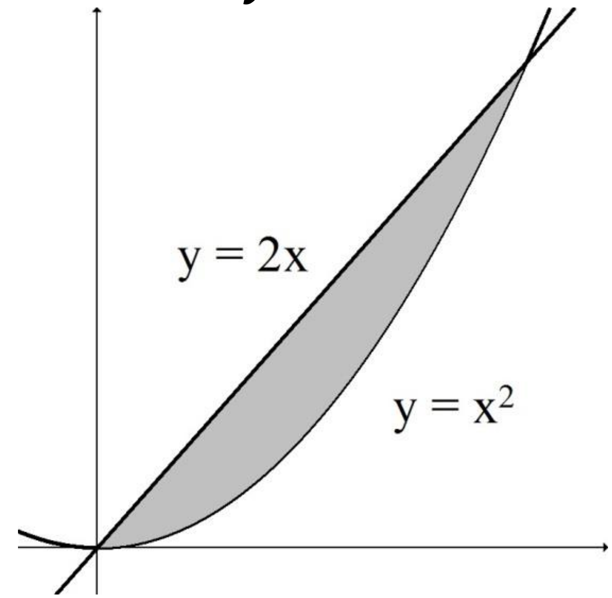




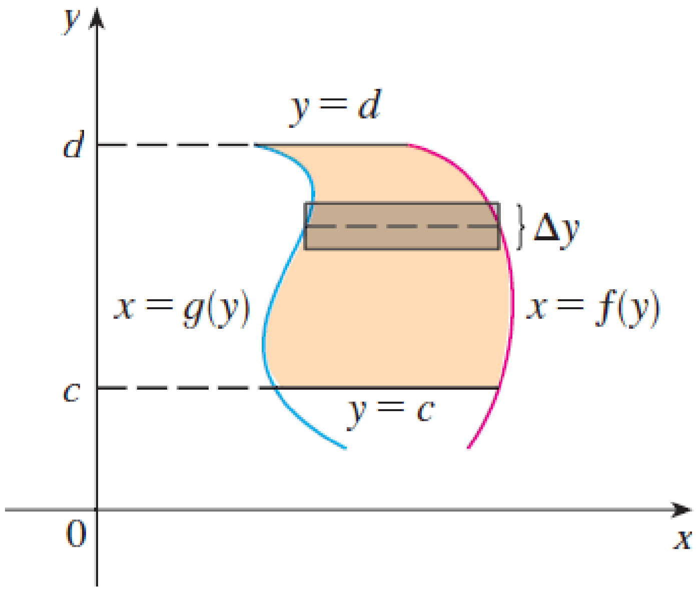
(a) Typical rectangle

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} (f(x_i) - g(x_i))\Delta x$$

Example: Find the area bounded between $y = 2x$ and $y = x^2$.

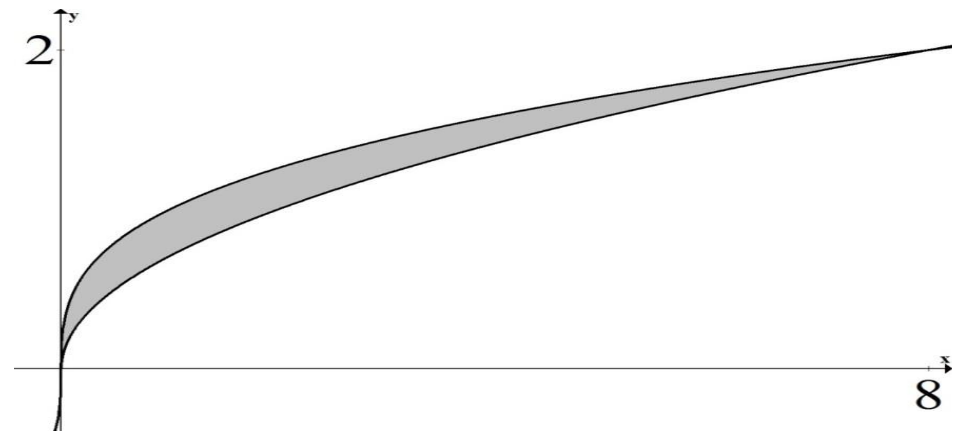


Using dy:



$$\text{Area} = \lim_{n\to\infty}\sum_{i=1}^{n}(f(y_i) - g(y_i))\Delta y$$

Example: Set up an integral for the area bounded between $x = 2y^2$ and $x = y^3$ (shown below) using dy.



Summary: The area between curves

1. Draw picture finding all intersections.

2. Choose dx or dy. Get ***everything*** in terms of the variable you choose.

3. Draw a typical approx. rectangle.

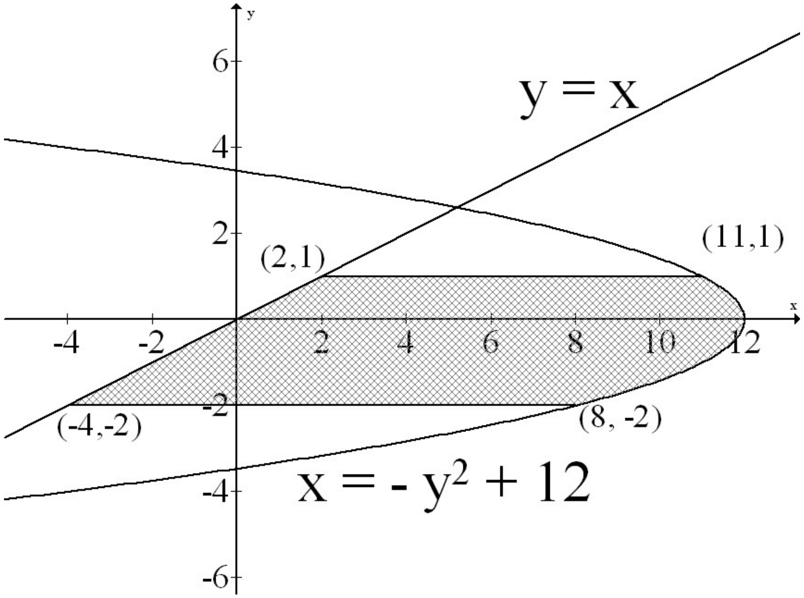
4. Set up as follows:

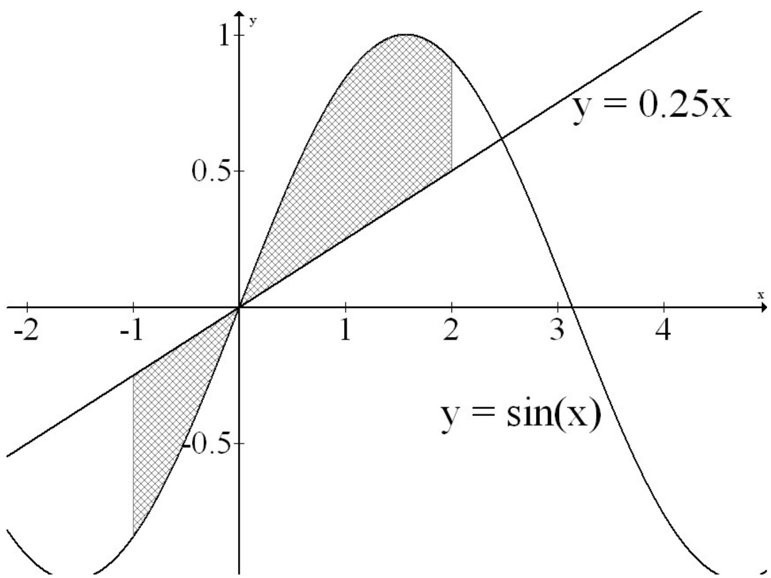
$$\text{Area} = \int_a^b (\text{TOP} - \text{BOTTOM})\, dx$$

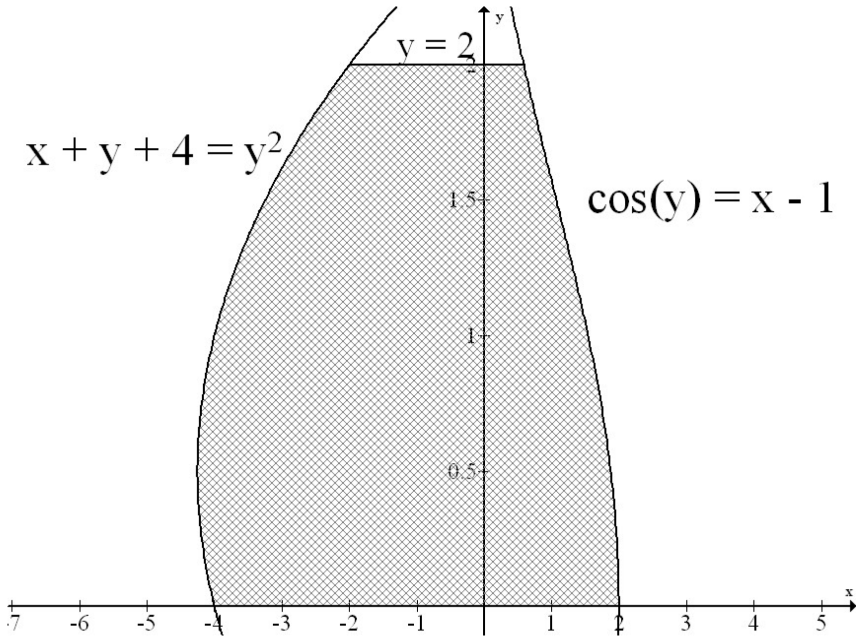
$$\text{Area} = \int_c^d (\text{RIGHT} - \text{LEFT})\, dy$$

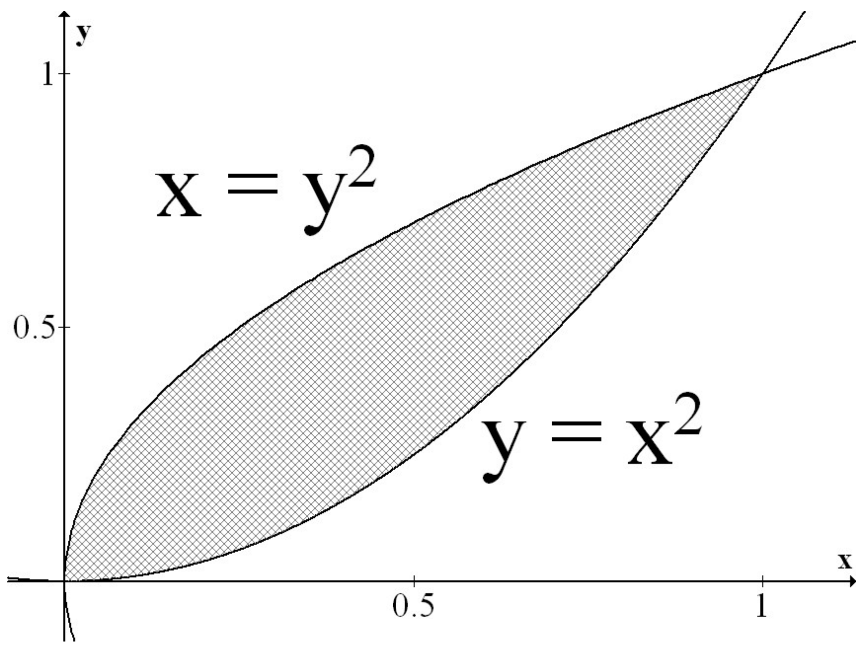
Example: Set up an integral (or integrals) that give the area of the region bounded by $x = y^2$ and $y = x - 2$.

Set up an integral for the total positive area of the following regions:






y = 2
x + y + 4 = y2
cos(y) = x - 1
y
x
-7
-6
-5
-4
-3
-2
-1
1
2
3
4
5
0.5
1
1.5


y
1
0.5
x = y2
y = x2
x
0.5
1